GENERATION OF INSTABILITY WAVES IN A BOUNDARY LAYER BY EXTERNAL TURBULENCE

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At the present time, due to numerous experimental and theoretical studies, the principles for calculation of critical Reynolds numbers for transition of a laminar boundary layer into a turbulent one have been formulated quite clearly on the basis of linear hydrodynamic stability theory. The possibility of employing linear theory to determine the "transition point" is based on the fact that at sufficiently small perturbation amplitudes in the flow, the transition begins with the development of so-called Tolmin–Schlichting waves, which can be described by linearized hydrodynamics equations. In this case a significant portion of the transition region is connected with those Tolmin–Schlichting waves, and the region of nonlinear perturbation development is relatively small [1]. Therefore, perturbation evolution in the boundary layer may be described with sufficient accuracy on the basis of linear theory, and the critical Reynolds number for transition may be determined approximately from the section in which the perturbation first reaches a threshold dimensionless amplitude value of $\varepsilon_* \sim 1\%$ at which the strongly nonlinear stage of development sets in [1]. However, the basic problem in development of a corresponding method of calculation at the present time is that of transformation of perturbations occurring under experimental conditions into Tolmin – Schlichting waves. In [2] the following possible mechanisms for excitation of Tolmin – Schlichting waves in a boundary layer were proposed: a) continuous generation over the entire extent of the boundary layer; b) generation in the vicinity of the model's forward edge; c) generation in the developed boundary layer through concentrated action. The mechanism most widely studied is that of Tolmin–Schlichting wave generation at the forward edge of the model. However, as the waves propagate down the flow to the point of stability loss they may damp out severely, so that their effect on the boundary layer transition will be insignificant. For such models, the mechanism of generation over the entire extent of the boundary layer is preferable. In [3] an analysis was performed of the interaction of a turbulent track of low intensity with an uncompressed boundary layer on a planar plate with the assumption of parallelness of the basic flow. The results showed that the perturbations considered penetrated the boundary layer only slightly and generation of Tolmin–Schlichting waves was absent. In the present study the same problem proposed in [3] will be analyzed, with consideration of a slight nonparallelism of the flow in the boundary layer. It will be shown that interaction of turbulent perturbations in the incident flow with Tolmin–Schlichting waves takes place, and analytical expressions will be obtained for determination of the intensity of Tolmin–Schlichting wave sources.

1. Formulation of the Problem. We will consider the problem of development of perturbations in an incompressible boundary layer on a plane plate within the framework of linearized Navier–Stokes equations. The boundary conditions will then be as follows: at y = 0 (on the plate surface) the adhesion condition is satisfied; at $x = x_0$ (in some section at a distance x_0 from the leading edge of the plate) perturbations u, v are specified (x and y are the components of the perturbation velocity) as functions of time; as $x \to \infty$ and $y \to \infty$ we impose the condition that the solution be finite. As usual, we turn to analysis of individual harmonics with real frequency ω; the basic equations take on the form

$$\begin{gathered} -i\omega u + U\frac{\partial u}{\partial x} + \quad \frac{\partial u}{\partial y} + u\frac{\partial U}{\partial x} + v\frac{\partial U}{\partial y} = -\frac{\partial p}{\partial x} + \nu\Delta u, \\ -i\omega v + U\frac{\partial v}{\partial x} + V\frac{\partial v}{\partial y} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} = -\frac{\partial p}{\partial y} + \nu\Delta v, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \end{gathered} \tag{1.1}$$

where U, V are the velocity components of the main flow (Blasius solution); ν is the kinematic viscosity coefficient; Δ is the Laplacian; p is the pressure perturbation referred to the density. If we now introduce the vector $\mathbf{A} = (A_1, A_2, A_3, A_4)$

$$A_1 = u,\ A_2 = p,\ A_3 = v,\ A_4 = \partial u/\partial y - \partial v/\partial x,$$

then the original system (1.1) may be written in the form

$$H_1\mathbf{A} = H_2\frac{\partial}{\partial x}\mathbf{A} + H_3\mathbf{A}, \tag{1.2}$$

where the operator H_3 is related to the terms containing V, $\partial U/\partial x$. In the given problem, it will be convenient to transform to boundary layer variables:

$$\xi = \frac{x}{x_0},\quad \eta = \frac{y}{\sqrt{\frac{\nu x}{U_0}}} = \frac{y}{R^{1/2}\xi^{1/2}}\frac{U_0}{\nu},\quad R = \frac{U_0 x_0}{\nu}, \tag{1.3}$$

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where U_0 is the velocity of the incident flow; x_0 is the coordinate of the initial section selected. Considering that the flow function of the main flow has the form

$$\psi = \nu \xi^{1/2} R^{1/2} f(\eta) + O(R^{-1/2}),$$

where $f(\eta)$ is a solution of the Blasius equation, we can obtain from Eq. (1.2) the following system of equations:

$$L_1 \mathbf{D} = \frac{\xi^{1/2}}{R^{1/2}} L_2 \frac{\partial}{\partial \xi} \mathbf{D} + \frac{\xi^{1/2}}{R^{1/2}} L_3 \mathbf{D}, \tag{1.4}$$

where the vector **D** is defined as

$$D_1 = \frac{A_1}{U_0}, \quad D_2 = \frac{A_2}{U_0^2}, \quad D_3 = \frac{A_3}{U_0}, \quad D_4 = A_4 \frac{\nu \xi^{1/2} R^{1/2}}{U_0^2},$$

and the operators L_1, L_2, L_3 have the form

$$L_1 = \begin{Vmatrix} i\beta & 0 & -f'' & \frac{1}{R^{1/2}\xi^{1/2}} \frac{\partial}{\partial \eta} \\ 0 & -\frac{\partial}{\partial \eta} & i\beta & 0 \\ 0 & 0 & -\frac{\partial}{\partial \eta} & 0 \\ \frac{\partial}{\partial \eta} & 0 & 0 & -1 \end{Vmatrix}, \quad L_2 = \begin{Vmatrix} f' & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R^{1/2}\xi^{1/2}} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{Vmatrix},$$

$$L_3 = \begin{Vmatrix} -\frac{\eta}{2\xi} f'' - \frac{1}{2\xi} f \frac{\partial}{\partial \eta} & -\frac{\eta}{2\xi} \frac{\partial}{\partial \eta} & 0 \\ 0 & 0 & \frac{\eta}{2\xi} f'' + \frac{1}{2\xi} (\eta f' - f) \frac{\partial}{\partial \eta} \\ -\frac{\eta}{2\xi} \frac{\partial}{\partial \eta} & 0 & 0 \\ 0 & 0 & -\frac{\eta}{2\xi} \frac{\partial}{\partial \eta} \end{Vmatrix}.$$

Here $\beta = \frac{\omega \nu}{U_0^2} R^{1/2} \xi^{1/2}$; and the prime denotes differentiation with respect to η. Since we are concerned only with terms of order $R^{-1/2}$, in the expression for L_3 we have omitted terms of higher order. The procedure used is analogous to that employed in [4].

2. **Eigenvectors.** Before constructing a solution for system (1.4), we will define the problem of constructing the eigenvector $\mathbf{A}_\alpha$,

$$L_1 \mathbf{A}_\alpha = i\alpha L_2 \mathbf{A}_\alpha, \tag{2.1}$$
$$\eta = 0 \;\; A_{\alpha 1} = A_{\alpha 3} = 0, \;\; \eta \to \infty \;\; |A_{\alpha i}| < \infty, \;\; i = 1, 2, 3, 4$$

and study the possible classes of eigenvectors. It can be shown that system (2.1) reduces to a single Orr–Sommerfeld equation for the components $A_{\alpha 3}$, which we will denote by $\Phi_0(\xi, \eta)$

$$(\alpha f' - \beta)\left(\Phi_0'' - \alpha^2 \Phi_0\right) - \alpha \Phi_0 f''' = \frac{1}{iR^{1/2}\xi^{1/2}} \left(\Phi_0^{IV} - 2\alpha^2 \Phi_0'' + \alpha^4 \Phi_0\right) \tag{2.2}$$

with boundary conditions

$$\eta = 0 \quad \Phi_0 = \Phi_0' = 0, \quad \eta \to \infty \quad |\Phi_0| < \infty, \tag{2.3}$$

where the prime denotes differentiation with respect to η and Φ_0 depends upon ξ as upon a parameter. The remaining components of the vector A_α are easily expressed in terms of Φ_0.

For $\eta \gg 1$ (outside the boundary layer), Eq. (2.2) transforms into an equation with constant coefficients and its general solution may be written as

$$\Phi_0 = C_1 e^{\alpha \eta} + C_2 e^{-\alpha \eta} + C_3 e^{\lambda \eta} + C_4 e^{-\lambda \eta},$$

where λ satisfies the equation

$$i(\alpha - \beta) = (1/R^{1/2}\xi^{1/2})(\lambda^2 - \alpha^2).$$

Now let φ_1, φ_2, φ_3, φ_4 be particular solutions of Eq. (2.2), having, as $\eta \to \infty$, the asymptotes $e^{\alpha\eta}$, $e^{-\alpha\eta}$, $e^{\lambda\eta}$, $e^{-\lambda\eta}$ respectively. These solutions are linearly independent, and the general solution of Eq. (2.2) has the form

$$\Phi_0 = C_1\varphi_1 + C_2\varphi_2 + C_3\varphi_3 + C_4\varphi_4.$$

We will now consider what requirements are imposed on the value of α and the special solutions of Eq. (2.2) by the boundary conditions (2.3). In all, five different cases are possible, corresponding to five possible classes of solutions of Eq. (2.2) with boundary conditions (2.3).

I) α is an imaginary number ($\alpha = i\gamma$, $\gamma > 0$). In view of the boundary conditions as $\eta \to \infty$ it is necessary to retain either φ_3 or φ_4, which correspond to exponential decay as $\eta \to \infty$. For definiteness, we take the branch of the square root, upon which Re $\lambda > 0$. Then it follows from Eq. (2.3) that $C_3 = 0$, and the general solution of Eq. (2.2) has the form

$$\Phi_0^{\mathrm{I}} = C_1^{\mathrm{I}}\varphi_1 + C_2^{\mathrm{I}}\varphi_2 + C_4^{\mathrm{I}}\varphi_4.$$

This solution must satisfy the following boundary conditions at $\eta = 0$, i.e.,

$$C_1^{\mathrm{I}}\varphi_1(\eta = 0) + C_2^{\mathrm{I}}\varphi_2(\eta = 0) + C_4^{\mathrm{I}}\varphi_4(\eta = 0) = 0,$$
$$C_1^{\mathrm{I}}\varphi_1'(\eta = 0) + C_2^{\mathrm{I}}\varphi_2'(\eta = 0) + C_4^{\mathrm{I}}\varphi_4'(\eta = 0) = 0.$$

Of this system of coefficients, C_1^{I}, C_2^{I}, C_4^{I} are defined to the accuracy of a constant multiplier, i.e., Φ_0 is defined uniquely to a normalized value. This class of solutions of Eq. (2.2) may be called pressure waves, in accordance with the classification presented in [5], since in this case as $\eta \to \infty$ for a plane-parallel approximation

$$\Phi_0^{\mathrm{I}} \sim \left(C_1^{\mathrm{I}} e^{i\gamma\eta} + C_2^{\mathrm{I}} e^{-i\gamma\eta}\right),$$

and if we calculate the dimensionless turbulence D_4, we find $D_4 = 0$, while the dimensionless pressure perturbation D_2 is found to be nonzero.

II) α is an imaginary number ($\alpha = i\gamma$, $\gamma < 0$). While the waves of class I solutions decay exponentially away from the boundary $x = x_0$ and correspond to the effect of this boundary, class II waves will correspond to exponential growth as $x \to \infty$ and the effect of the righthand boundary in problems having such a boundary located at finite $x = L$. Waves of this class propagate leftward.

III, IV) $\lambda = i\gamma$, γ is real, $0 < \gamma < \infty$. In this case α is defined as

$$i\alpha_{\mathrm{III,IV}} = \frac{R^{1/2}\xi^{1/2}}{2}\left[1 \mp \sqrt{1 + \frac{4\left(\frac{\gamma^2}{R^{1/2}\xi^{1/2}} - i\beta\right)}{R^{1/2}\xi^{1/2}}}\right].$$

Class III waves correspond to a minus sign in the expression for α. For $R^{1/2}\xi^{1/2} \to \infty$ $i\alpha_{\mathrm{III}} \approx -\gamma^2/R^{1/2}\xi^{1/2} + i\beta$ waves of this class in the planoparallel approximation correspond to a solution of Eq. (1.4) in the form of waves propagating down the flow and weakly damping with distance from the initial section $x = x_0$.

The plus sign in the expression for α corresponds to class IV waves, and for $R^{1/2}\xi^{1/2} \to \infty$ $i\alpha_{\mathrm{IV}} \approx R^{1/2}\xi^{1/2} - i\beta$. In the planoparallel approximation class IV corresponds to a solution of Eq. (1.4) in the form of waves propagating up the flow, manifesting the effect of a boundary at $x = L$ (if such occurs in the problem formulated).

Class III and IV waves can be related to the turbulence waves of [5], since as $\eta \to \infty$ in the plane-parallel approximation $D_2 = 0$, $D_4 \neq 0$. In the literature class III waves are also termed removal waves.

V) α is complex, and λ is complex, Re $\alpha > 0$, Re $\lambda \neq 0$.

As an example, we take Re $\alpha > 0$ and Re $\lambda > 0$ (the other possible signs of Re α and Re λ are treated in a similar manner).

In this case the solution of Eq. (2.2) has the form

$$\Phi_0^{\mathrm{V}} = C_2^{\mathrm{V}}\varphi_2 + C_4^{\mathrm{V}}\varphi_4,$$

and from the boundary conditions at $\eta = 0$ we obtain

$$C_2^{\mathrm{V}}\varphi_2(\eta = 0) + C_4^{\mathrm{V}}\varphi_4(\eta = 0) = 0, \qquad (2.4)$$
$$C_2^{\mathrm{V}}\varphi_2'(\eta = 0) + C_4^{\mathrm{V}}\varphi_4'(\eta = 0) = 0.$$

The solubility condition for Eq. (2.4) will be

$$\varphi_2(\eta=0)\cdot\varphi_4'(\eta=0)-\varphi_2'(\eta=0)\cdot\varphi_4(\eta=0)=0. \tag{2.5}$$

Equation (2.5) is a transcendental equation for determination of α and may, depending on the value of $R\xi$, either have no solution in general, have a finite number of solutions, or have an exact set of solutions. To each root α of Eq. (2.5) there corresponds an Orr–Sommerfeld solution of Eq. (2.2), defined to the accuracy of normalization. The question of the spectrum of solutions of Eq. (2.5) in the case of flow in a boundary layer is still unanswered at present, and will not be considered herein. Class V solutions correspond to Tolmin–Schlichting waves, which can either grow or decay down the flow (depending on the sign of $\mathrm{Im}\alpha$). Solutions of Eq. (2.5) with $\mathrm{Re}\,\alpha<0$ would then correspond to waves propagating up the flow. However, waves of this class with $\mathrm{Re}\,\alpha<0$ have not yet been detected in flows in a boundary layer.

We will now define the product of the two eigenvectors $\mathbf{A}_\alpha$, $\mathbf{A}_\gamma$ of the classes considered above as

$$(\mathbf{A}_\alpha,\ \mathbf{A}_\gamma)=\lim_{\varepsilon\to 0}\int_0^\infty\sum_{i=1}^4 A_{\alpha i}A_{\gamma i}\exp(-\varepsilon\eta)\,d\eta,$$

where $\epsilon>0$. We may then substitute in the conjugate problem corresponding to Eq. (2.1)

$$L_1^*\mathbf{B}_\gamma=i\gamma L_2^*\mathbf{B}_\gamma,$$
$$\eta=0\ B_{\gamma 2}=B_{\gamma 4},\ \eta\to\infty\ |B_{\gamma i}|<\infty,\ i=1,\ 2,\ 3,\ 4,$$

where L_1^*, L_2^* are operators conjugate to L_1 and L_2, defined by the condition $(\mathbf{LA},\mathbf{B})=(\mathbf{A},\mathbf{L^*B})$. It can easily be shown that if $\alpha\neq\gamma$, then $(L_2\mathbf{A}_\alpha,\ \mathbf{B}_\gamma)=0$.

3. Construction of a Solution to System (1.4). In constructing approximate solutions to system (1.4) we will limit ourselves to the following problem: Let a turbulence wave propagate down the flow with frequency ω and specified amplitude at $x = x_0$. Due to inhomogeneity of the flow in the x-direction [the operator L_3 in Eq. (1.4)] wave of all classes I-V will develop in the flow. These waves will interact with the main flow and each other. In the present study this interaction will be taken as a model of the interaction of turbulence of the incident flow, in which a turbulence wave can always be distinguished, with waves in the boundary layer. For practical purposes, the most interesting of these modes is formation of Tolmin–Schlichting waves, which alone of the five wave classes can grow in the direction of their own propagation, and can in principle reach the amplitude of the original turbulence wave or even exceed it. Because of the weakness of interaction between the various modes, in a first approximation we may retain only the original turbulence wave and the Tolmin–Schlichting waves, neglecting the effect of the remaining waves on these two modes.

Construction of the eigenvector $\mathbf{A}_{Ts}$ corresponding to a Tolmin–Schlichting wave reduces to solution of the Orr–Sommerfeld equation (2.2) for $\mathbf{A}_{Ts3}$ with boundary conditions

$$\eta=0\quad \Phi_{0Ts}=\Phi_{0Ts}'=0,\ \eta\to\infty\quad \Phi_{0Ts},\quad \Phi_{0Ts}'\to 0.$$

The eigenvalue α is then determined from the condition of solubility of the boundary problem (Eq. (2.5)) and depends upon ξ as upon a parameter. Construction of the eigenvector $\mathbf{A}_v$ corresponding to the turbulence wave (class III) reduces to construction of three linearly independent solutions of Eq. (2.2) with asymptote $e^{\pm i\gamma\eta}, e^{-\beta\eta}$ (since $\alpha_{III}\approx\beta$ for $R^{1/2}\xi^{1/2}\to\infty$) and to determination of coefficients for each independent solution from the condition that the general solution of Eq. (2.2) satisfy the boundary condition at $\eta = 0$. If we seek a solution of Eq. (1.4) in the form

$$\mathbf{D}=C_{Ts}(\xi)\mathbf{A}_{Ts}(\alpha,\ \eta,\ \xi)+C_v(\xi)\mathbf{A}_v(\gamma,\ \eta,\ \xi),$$

then we obtain for determination of C_{TS} and C_v the following system of equations:

$$\frac{\partial C_{Ts}}{\partial\xi}-\frac{i\alpha R^{1/2}}{\xi^{1/2}}C_{Ts}+C_{Ts}\frac{\left[\left(L_2\frac{\partial\mathbf{A}_{Ts}}{\partial\xi},\ \mathbf{B}_{Ts}\right)+(L_3\mathbf{A}_{Ts},\ \mathbf{B}_{Ts})\right]}{(\mathbf{A}_{Ts},\mathbf{B}_{Ts})}+\frac{C_v\left[\left(L_2\frac{\partial\mathbf{A}_v}{\partial\xi},\ \mathbf{B}_{Ts}\right)+(L_3\mathbf{A}_v,\ \mathbf{B}_{Ts})\right]}{(\mathbf{A}_{Ts},\mathbf{B}_{Ts})}=0; \tag{3.1a}$$

$$\frac{\partial C_v}{\partial\xi}-i\frac{\beta R^{1/2}}{\xi^{1/2}}C_v+\frac{C_v\left[\left(L_2\frac{\partial\mathbf{A}_v}{\partial\xi},\ \mathbf{B}_v\right)+(L_3\mathbf{A}_v,\ \mathbf{B}_v)\right]}{(\mathbf{A}_v,\mathbf{B}_v)}+C_v\frac{\left[\left(L_2\frac{\partial\mathbf{A}_{Ts}}{\partial\xi},\ \mathbf{B}_v\right)+(L_3\mathbf{A}_{Ts},\ \mathbf{B}_v)\right]}{(\mathbf{A}_v,\mathbf{B}_v)}=0. \tag{3.1b}$$

Construction of the vector $\mathbf{B}_{TS}$ reduces to solution of the conjugate Orr–Sommerfeld equation for the components $\mathbf{B}_{TS2}$ which we denote by χ:

$$(\alpha f'-\beta)(\chi''-\alpha^2\chi)+2\alpha\chi'f''=\frac{1}{iR^{1/2}\xi^{1/2}}(\chi^{IV}-2\alpha^2\chi''+\alpha^4\chi) \tag{3.2}$$

with boundary conditions

$$\eta=0\ \chi=\chi'=0,\ \eta\to\infty\ \chi,\ \chi'\to 0.$$

Construction of the vector $\mathbf{B}_v$ also reduces to solution of Eq. (3.2) for $\alpha = \alpha_{III}$. To do this it is necessary to construct particular solutions of Eq. (3.2) with the same asymptotes as in the construction of the vector $\mathbf{A}_v$, and then to use the boundary conditions at $\eta = 0$ to define the coefficients with which these particular solutions appear in the general solution of Eq. (3.2).

The solution of Eq. (3.1a) with $C_v \equiv 0$ reduces to consideration of the effect of slight nonparallelism of the flow in the boundary layer upon its stability, and was considered in [4]. The terms containing C_v in Eq. (3.1a) represent the generation of Tolmin–Schlichting waves by the turbulence of the incident flow, continuously distributed over the boundary layer. If x is not too large we may neglect the interaction $C_{TS} \to C_v$ and assume that

$$C_v(\xi) = \varepsilon_0(\omega)\, e^{i\int_1^{\xi} \frac{R^{1/2}\beta}{\xi^{1/2}} d\xi} = \varepsilon_0(\omega)\, e^{i\frac{\omega(x-x_0)}{U_0}},$$

where $\epsilon_0(\omega)$ corresponds to the amplitude of the incident flow perturbation with frequency ω. In this approximation one can calculate the intensity of Tolmin–Schlichting wave sources as

$$q_i(\omega, \xi) = -\frac{C_v(\xi)\left[\left(L_2\frac{\partial \mathbf{A}_v}{\partial \xi}, \mathbf{B}_{TS}\right) + (L_3\mathbf{A}_v, \mathbf{B}_{TS})\right]}{(\mathbf{A}_{TS}, \mathbf{B}_{TS})}.$$

Since at small x outside the instability region Tolmin–Schlichting waves damp out strongly, their effect on the boundary layer transition may be neglected, i.e., we may choose a section x_0, in which the approximation $C_{TS}(1) = 0$ is applicable. Then, calculating $C_{TS}(\xi)$ from Eq. (3.1a) with a known background of perturbations in the incident flow, and using the fact that the nonlinear regime of flow development sets in at a velocity pulsation amplitude of the order of magnitude of 1% of the incident flow velocity [1], we may approximately determine at what frequency and in what section the amplitude of the Tolmin-Schlichting wave reaches the critical value ϵ_*, and thus calculate the dependence of the transition Reynolds number of the original turbulence.

In conclusion, we note that the above mechanism of Tolmin–Schlichting wave generation by incident flow turbulence has a simple physical meaning. Since the flow lines of the main flow penetrate the boundary layer through its outer boundary, convection of turbulent perturbations propagating along the flow lines occurs within the boundary layer, where these perturbations are converted into Tolmin–Schlichting waves. This convection mechanism will then be most effective at low Reynolds numbers (in the vicinity of the turbulence), although the related generation of Tolmin–Schlichting waves will not affect the transition to turbulence because of intense damping of perturbations at low Reynolds numbers. On the other hand, the highest Tolmin–Schlichting wave amplification coefficients, observed in the low-frequency part of the spectrum, occur only at high Reynolds numbers, where the mechanism of convection of external perturbations within the boundary layer proves ineffective. Consequently, there exists a definite interval of Reynolds numbers, within which transformation of incident flow turbulence into Tolmin–Schlichting waves is the defining factor in transition to turbulence.

It should be noted that the method presented above can be expanded in the very same manner to supersonic flows, in which for one frequency there may exist several nondamping modes, so that it will be necessary to also consider interaction of these modes among themselves.

In the present study the interaction between incident flow turbulence and Tolmin–Schlichting waves was produced by nonparallelness of the flow. In [6, 7] a weak compressibility of the flow was considered, and the possibility of Tolmin–Schlichting wave generation by acoustical perturbations on the roughness of the surface flowed over was analyzed.

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